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INTUITIONISTIC FUZZY SOFT GRAPH STRUCTURES

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Abstract: Fuzzy sets (IFS) introduced by Zadeh, in 1965 has various applications in almost all fields of science and technology .In fuzzy set theory, membership of an element to a fuzzy set is a single value between zero and one. A generalization of fuzzy set was proposed by Atanassov as intuitionistic fuzzy sets (IFS) which includes the degree of hesitation. The notion of defining intuitionistic fuzzy set as generalized fuzzy set is useful in many application areas. In 1975 Rosenfeld introduced the concept of fuzzy graphs. The fuzzy relations between fuzzy sets were also considered by Rosenfield . He developed the structure of fuzzy graphs. This has analogy with several graph theoretical concepts. Bhattacharya gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and Peng. In this paper, we extended these to define intuitionistic fuzzy soft graphs.

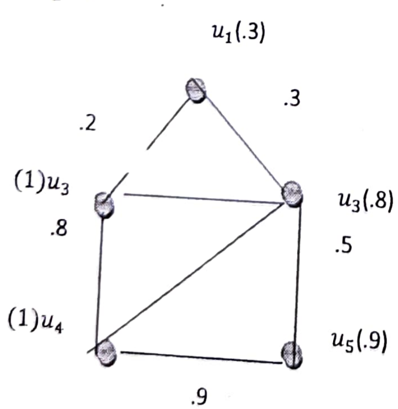
Key words: Soft set, Fuzzy soft set, intuitionistic fuzzy soft graphs.

1. Preliminaries

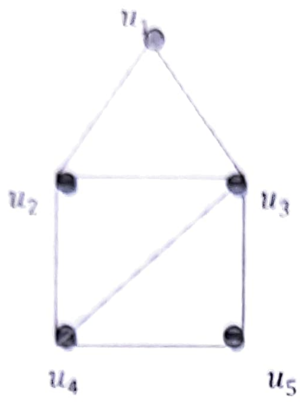
Let V be a non empty set. A fuzzy graph is a pair of functions G: (σ, μ) where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation[4] on σ. i.e. $\sigma : V \to [0, 1]$ and $\mu: V \times V \to [0, 1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all u, v in V. We denote the underlying (crisp) graph of G: (σ, μ) by G* $:(\sigma^*, \mu^*)$ where σ^* is referred to as the (nonempty) set V of nodes and $\mu^* = E \subseteq V \times V$. Note that the crisp graph (V, E) is a special case of a fuzzy graph with each vertex and edge of (V, E) having degree of membership 1. We need not consider loops and we assume that μ is reflexive.

1.1. Definition. The fuzzy graph $H: (\tau, \nu)$ is called a partial fuzzy subgraph [4] of G : (σ, μ) if $\tau \subseteq \sigma$ and $\nu \subseteq \mu$. In particular, we call $H: (\tau, \nu)$ a fuzzy subgraph of G: (σ, μ) if $\tau(u) = \sigma(u)$ $\forall u \square \tau^*$ and $\nu(u, v) = \mu(u, v)$ $\forall (u, v) \square \nu^*$.

1.2. Example.



A Fuzzy graph G



Graph G^*

A fuzzy graph G: (σ, μ) is strong if $\mu(u,v) = \sigma(u) \wedge \sigma(v) \forall (u, v)$ □, μ^* and is complete if. $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ ∀u, v□ σ^* .

1.2. Definition. The complement of a fuzzy graph G : (σ, μ) is the fuzzy graph $\bar{G}$: $(\bar{\sigma}, \bar{\mu})$ where $\bar{\sigma} \equiv \sigma$ and $\bar{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v)$ for all u, v in V[4].

1.3. Definition. An intuitionistic fuzzy graph [1],[2] with underlying set V is defined to be a pair G = (A, B) where

(i) the functions μ_A: V→ [0, 1] and ν_A: V → [0, 1] denote the degree of membership and non-membership of the element x □V, respectively such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all x □V

(ii) the functions μ_B: E ⊆ V x V → [0, 1] and ν_B: E ⊆ V x V → [0, 1] are defined by $\mu_B\{x, y\} \leq \min(\mu_A(x)\mu_A(y))$ and $\nu_B\{x,y\} \geq \max(\nu_A(x); \nu_A(y))$ such that $0 \leq \mu_B(\{x, y\}) + \nu_B(\{x,y\}) \leq 1$ for all {x, y} □ E.

We call *A* the intuitionistic fuzzy vertex set of V, *B* the intuitionistic fuzzy edge set of G, respectively. Note that *B* is a symmetric intuitionistic fuzzy relation on *A*. We use the notation xy for an element of E. Thus, G = (A, B) is an intuitionistic graph of G^* = (V, E) if $\mu_B(xy) \leq \min(\mu_A(x), \mu_{AB}(y))$ and $\nu_B(xy) \geq \max(\nu_A(x), \nu_A(y))$ for all (xy) □ E.

1.4. Definition. An intuitionistic fuzzy graph G = (A,B) is called strong intuitionistic fuzzy graph if

$\mu_B(xy) = \min(\mu_A(x), \mu_A(y))$ and $\nu_B(xy) = \max(\nu_A(x), \nu_A(y))$ for all (xy) □ E.

1.5. Definition. Let A = (μ_A, ν_A) and $A' = (\mu_A', \nu_A')$ be intuitionistic fuzzy subsets of V_1 and V_2 and let B = (μ_B, ν_B) and $B' = (\mu_B', \nu_B')$ be intuitionistic fuzzy subsets of E_1 and E_2, respectively. The Cartesian product of two strong intuitionistic fuzzy graphs G_1 and G_2 of the graphs G_1^* and G_2^* is denoted by $G_1 \times G_2 = (A \times A', B \times B')$ and is defined as follows:

1. $(\mu_A \times \mu_A')(x_1, x_2) = \min(\mu_A(x_1), \mu_A'(x_2))$
$(\nu_A \times \nu_A')(x_1, x_2) = \max(\nu_A(x_1), \nu_A'(x_2))$ For all (x_1, x_2) □ V,
2. $(\mu_B \times \mu_B')((x, x_2)(x, y_2)) = \min(\mu_A(x), \mu_B'(x_2 y_2))$
$(\nu_B \times \nu_B')(x, x_2)(x, y_2) = \max(\nu_A(x), \nu_B'(x_2 y_2))$ For all x□ V_1, for all $x_2 y_2$□ E_2,
3. $(\mu_B \times \mu_B')((x_1, z)(y_1, z)) = \min(\mu_B(x_1 y_1), \mu_A'(z))$
$(\nu_B \times \nu_B')(x_1, z)(y_1, z) = \max(\nu_B(x_1 y_1), \nu_A'(z))$ For all z□ V_2, for all $x_1 y_1$□ E

2. Fuzzy Soft Graph

2.1. Definition. Let $A = \{x_1, x_2, \ldots . x_n\}$, E be the set of parameters and $A \subseteq E$.

Also let i) $\rho: A \to F(V)$ (collection of all fuzzy subsets in V) where $e \mapsto \rho(e) = \rho_e(say)$and $\rho_e: V \to [0,1]$ where $x \mapsto \rho_e(x_i)$then (A, ρ) is called Fuzzy soft vertex.

2.2. Definition. (Fuzzy Soft edge). Let $= \{x_1, x_2, \dots. x_n\}$, E be the set of parameters and $A \subseteq E$. Also let i) $\mu: A \to F(V \times V)(\,collection\ of\ all\ fuzzy\ subsets\ in\ V \times V)$ where $e \mapsto \mu(e) = \mu_e(say)$and $\mu_e: V \times V \to [0,1]$ where $x_i, x_j \mapsto \mu_e(x_i, x_j)$then (A, μ) is called Fuzzy soft edge.

$((A, \rho)(A, \mu))$is called a fuzzy soft graph iff $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ for all $e \in A\ and\ for\ all\ i, j = 1,2, \dots n$ denoted by$G_{A,V}$

2.3. Definition. (Fuzzy soft subgraph). The fuzzy soft sub graph $H_{A,V} = ((A, \tau), (A, \upsilon))$ is called a fuzzy soft sub graph of

$$G_{A,V} = ((A, \rho), (A, \mu))if\ \rho_e(x_i) \geq \tau_e(x_i) forall\ x_i \epsilon\ V, e \epsilon A$$

2.4. Definition. (Spanning fuzzy Soft graph). The fuzzy soft subgraph$H_{A,V} = ((A, \tau), (A, \upsilon))$ is said to be a spanning fuzzy soft subgraph of $G_{A,V} = ((A, \rho), (A, \mu))if\ \rho_e(x_i) = \tau_e(x_i) forall\ x_i \epsilon\ V, e \epsilon A$

2.5. Definition.(Underlying Crisp Graph). The Underlying Crisp Graph of a fuzzy soft graph $G_{A,V} = ((A, \rho), (A, \mu))$ is denoted by $G^* = (\rho^*, \mu^*)$ where $\rho^* = \{x_i \epsilon V \,/\, \rho_e(x_i) > 0 for\ some\ e \epsilon E\}$ and

$$\mu^* = \{(x_i, x_j) \epsilon V \times V \,/\, \mu_e(x_i, x_j) > 0 for\ some\ e \epsilon E\}$$

2.6. Definition.(Strong fuzzy soft graph).A fuzzy soft graph $G_{A,V} = ((A, \rho), (A, \mu))$ is called a strong fuzzy soft graph if $\mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j)$for all $(x_i, x_j) \epsilon \mu^*, e \epsilon A$

2.7. Definition. (Complete fuzzy soft graph). A fuzzy soft graph is said to be complete if $\mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j)$ for all $x_i, x_j \epsilon \rho^*, e \epsilon A$.

In the next section we extend the soft fuzzy graph into intutionistic fuzzy soft graph.

3. Intuitionistic fuzzy soft graph

3.1.Definition.(Intutionistic fuzzy soft vertex). Let $= \{x_1, x_2, \ldots . x_n\}$, $E = \{e_1, e_2, \ldots e_n\}$ be the set of parameters and $A \subseteq E$. Also let i) $(\rho_1, \rho_2): A \rightarrow F(V)(\textit{collection of all fuzzy subsets in } V)$where$\rho_1: A \rightarrow F(V)$ such that $e \mapsto \rho_1(e) = \rho_1{}^e\ (say)$and $\rho_1{}^e: V \rightarrow [0,1]$. Also $\rho_2: A \rightarrow F(V)$ such that $e \mapsto \rho_2(e) = \rho_2{}^e\ (say)and\rho_2{}^e: V \rightarrow [0,1]\ x_i \mapsto \rho_1{}^e(x_i) and\ x_i \mapsto \rho_2{}^e(x_i)\ such\ that\ 0 \le \rho_1{}^e(x_i) + \rho_2{}^e(x_i) \le 1\ \forall\ x_i \in V.$ Then (A, ρ_1, ρ_2) is called Intutionistic Fuzzy soft vertex.

3.2. Definition. (Intutionistic Fuzzy Soft edge). Let $= \{x_1, x_2, \ldots . x_n\}$, $E = \{e_1, e_2, \ldots e_n\}$ be the set of parameters and $A \subseteq E$. Also let i) $(\mu_1, \mu_2): A \rightarrow F(V \times V)(\textit{collection of all fuzzy subsets in } V)$where$\mu_1: A \rightarrow F(V \times V)$ such that $e \mapsto \mu_1(e) = \mu_1{}^e\ (say)$and $\mu_1{}^e: V \times V \rightarrow [0,1]$. Also $\mu_2: A \rightarrow F(V \times V)$ such that $e \mapsto \mu_2(e) = \mu_2{}^e\ (say)and\mu_2{}^e: V \times V \rightarrow [0,1\ (x_i, x_j) \mapsto \mu_1{}^e(x_i, x_j)\ and\ (x_i, x_j) \mapsto \mu_2{}^e(x_i, x_j)\ such\ that\ 0 \le \mu_1{}^e(x_i x_j) + \mu_2{}^e(x_i x_j) \le 1\ \forall\ x_i x_j \in V \times V$. Then (A, μ_1, μ_2) is called Intutionistic Fuzzy soft edge

3.3. Definition. Intutionistic Fuzzy Soft Graph. $((A, \rho_1, \rho_2), (A, \mu_1, \mu_2))$ is called intutionistic fuzzy soft graph iff$\mu^e{}_1(x_i, x_j) \le \rho_1{}^e(x_i) \wedge \rho_1{}^e(x_j)\ \forall\ e\epsilon A, \forall x_i, x_j \in V$and $\mu^e{}_2(x_i, x_j) \le$

$\rho_2{}^e(x_i) V \rho_2{}^e(x_j)\ \forall\, e\epsilon A,\ \forall x_i, x_j \in V$. Also, $0 \le \mu_1{}^e(x_i) + \mu_2{}^e(x_i) \le 1\ \forall\, x_i, x_j \epsilon V \times V$. An Intutionistic Fuzzy Soft graph is denoted by $IG_{A,V}$

3.4. Definition. Intutionistic fuzzy soft subgraph. The Intutionistic fuzzy soft graph $IH_{A,V} = ((A, \rho_1{}^*, \rho_2{}^*), (A, \mu_1{}^*, \mu_2{}^*))$ is called an Intutionistic fuzzy soft sub graph of $IG_{A,V} = ((A, \rho_1, \rho_2), (A, \mu_1, \mu_2))$ if $\rho_1{}^e(x_i) \ge \rho_1{}^{*e}(x_i)$ and $\rho_2{}^e(x_i) \le \rho_2{}^{*e}(x_i)$ for all $x_i \epsilon V, e\epsilon A$.

Also, $\mu^{*e}{}_1(x_i, x_j) \le \mu^e{}_1(x_i, x_j), \mu^e{}_2(x_i, x_j) \le \mu^{*e}{}_2(x_i, x_j)$,

3.5. Definition. (Spanning IFS Subgraph). The IFS subgraph $IH_{A,V} = ((A, \rho_1{}^*, \rho_2{}^*), (A, \mu_1{}^*, \mu_2{}^*))$ is said to be a spanning IFS subgraph of $IG_{A,V} = ((A, \rho_1, \rho_2), (A, \mu_1, \mu_2))$ if

$\rho_1{}^e(x_i) = \rho_1{}^{*e}(x_i)$ and $\rho_2{}^e(x_i) = \rho_2{}^{*e}(x_i)$ for all $x_i \epsilon V, e\epsilon A$

3.6. Definition. (Underlying Crisp Graph). The underlying crisp graph of a IFSG $IG_{A,V} = ((A, \rho_1, \rho_2), (A, \mu_1, \mu_2))$ is denoted by $G^* = (\widetilde{\rho_1}, \widetilde{\rho_2}, \widetilde{\mu_1}, \widetilde{\mu_2})$ where $\tilde{\rho}$ is $\{\widetilde{\rho_1} = \{x_i \epsilon V / \rho_1{}^e(x_i) > 0$ for some $e\epsilon A\}$ and $\{\widetilde{\rho_2} = \{x_i \epsilon V / \rho_2{}^e(x_i) > 0$ for some $e\epsilon A\}$.

Also, $\tilde{\mu}$ is $\{\widetilde{\mu_1} = \{(x_i, x_j) \epsilon V \times V / \mu^e{}_1(x_i, x_j) > 0\ , e\epsilon A\}$,

$\widetilde{\mu_2} = \{(x_i, x_j) \epsilon V \times V / \mu^e{}_2(x_i, x_j) > 0\ , e\epsilon A\}$

An IFSG is denoted by $IG_{A,V} = ((A, \rho_1, \rho_2), (A, \mu_1, \mu_2)$

3.7. Definition. (Strong intutionistic fuzzy soft graph). An IFSG $IG_{A,V} = ((A, \rho_1, \rho_2), (A, \mu_1, \mu_2))$ is called a strong IFSG if $\mu^e{}_1(x_i, x_j) = \rho_1{}^e(x_i) \wedge \rho_1{}^e(x_j)$ and $\mu^e{}_2(x_i, x_j) = \rho_2{}^e(x_i) \wedge \rho_2{}^e(x_j), \forall\, e\epsilon A, \forall (x_i, x_j) \widetilde{\mu_1}, \widetilde{\mu_2} \epsilon V$

3.8. Definition. (Union and Intersection). Let $IG_{A,V} = ((A, \rho_1, \rho_2)$, $(A, \mu_1, \mu_2))$and $IG_{B,V} = ((B, \rho_1, \rho_2), (B, \mu_1, \mu_2))$ be two IFSGs. Then

(i) A $\subseteq$ B if and only if $\rho_{1_A}(x_i) \leq \rho_{1_B}(x_i)$ an $\rho_{2_A}(x_i) \leq \rho_{2_B}(x_i), \forall x_i \in V$.

Also $\mu_{1_A}(x_i) \leq \mu_{1_B}(x_i)$ and $\mu_{2_A}(x_i) \leq \mu_{2_B}(x_i), \forall x_i \in V$.

(ii) A = B if and only if $\rho_{1_A}(x_i) = \rho_{1_B}(x_i)$ an $\rho_{2_A}(x_i) = \rho_{2_B}(x_i), \forall x_i \in V$.

Also $\mu_{1_A}(x_i) = \mu_{1_B}(x_i)$ and $\mu_{2_A}(x_i) = \mu_{2_B}(x_i), \forall x_i \in V$.

(iii) A ∩ B = {< x, $(\rho_{1_A} \cap \rho_{1_B})(x)$; $(\rho_{2_A} \cap \rho_{2_B})(x)$ $(\mu_{1_A} \cap \mu_{1_B})(x)$, $(\mu_{2_A}$ $\mu_{2_B})(x)$, > : x □X,} , where

$(\rho_{1_A} \cap \rho_{1_B})(x) = \text{Min}\{\rho_{1_A}(x), \rho_{1_B}(x)\} = \rho_{1_A}(x) \wedge \rho_{1_B}(x)$,

$(\rho_{2_A} \cap \rho_{2_B})(x) = \text{Min}\{\rho_{2_A}(x), \rho_{2_B}(x)\} = \rho_{2_A}(x) \wedge \rho_{2_B}(x)$ and

$(\mu_{1_A} \cap \mu_{1_B})(x) = \text{Max}\{\mu_{1_A}(x), \mu_{1_B}(x)\} = \mu_{1_A}(x) \vee \mu_{1_B}(x)$,

$(\mu_{2_A} \cap \mu_{2_B})(x) = \text{Max}\{\mu_{2_A}(x), \mu_{2_B}(x)\} = \mu_{2_A}(x) \vee \mu_{2_B}(x)$

(iv) A∪B ={< x,$(\rho_{1_A} \cup \rho_{1_B})(x)$,$(\rho_{2_A} \cup \rho_{2_B})(x)$,$(\mu_{1_A} \cup \mu_{1_B})(x)$,$(\mu_{2_A} \cup$ $\mu_{2_B})(x)$,>: x □X,} , where $(\rho_{1_A} \cup \rho_{1_B})(x) = \text{Max}\{\rho_{1_A}(x), \rho_{1_B}(x)\} = \rho_{1_A}(x)$ $\rho_{1_B}(x)$,

$(\rho_{2_A} \cap \rho_{2_B})(x) = \text{Max}\{\rho_{2_A}(x), \rho_{2_B}(x)\} = \rho_{2_A}(x) \vee \rho_{2_B}(x)$ and

$(\mu_{1_A} \cup \mu_{1_B})(x) = \text{Min}\{\mu_{1_A}(x), \mu_{1_B}(x)\} = \mu_{1_A}(x) \wedge \mu_{1_B}(x)$,

$(\mu_{2_A} \cup \mu_{2_B})(x) = \text{Min}\{\mu_{2_A}(x), \mu_{2_B}(x)\} = \mu_{2_A}(x) \wedge \mu_{2_B}(x)$

3.9. Definition. (Product IFSG). An IFSG is called a product IFSG if $\mu^{e}{}_{1}(x_i, x_j) \leq \rho_1{}^{e}(x_i) \times \rho_1{}^{e}(x_j)$ and $\mu^{e}{}_{2}(x_i, x_j) \leq \rho_2{}^{e}(x_i) \times \rho_2{}^{e}(x_j), \forall\, e\epsilon A, \forall x_i, x_j \epsilon V$ where × defines a usual product on fuzzy set.

3.10. Definition. (Complete product IFSG). A product IFSG is called complete product IFSG if $\mu^{e}{}_{1}(x_i, x_j) = \rho_1{}^{e}(x_i) \times \rho_1{}^{e}(x_j)$ and $\mu^{e}{}_{2}(x_i, x_j) = \rho_2{}^{e}(x_i) \times \rho_2{}^{e}(x_j), \forall\, e\epsilon A, \forall x_i, x_j \epsilon V$

If $\rho_1{}^{e}, \rho_2{}^{e}, \mu^{e}{}_{1}$ *and* $\mu^{e}{}_{2}$ are normal then

$(\mu^{e}{}_{1})^{n}(x_i, x_j) = \vee_{x_i, x_j \epsilon V}\{(\mu_1{}^{e})^{n-1}(x_i, x_j) \times \mu_1{}^{e}(x_i, x_j)\}$ and $(\mu^{e}{}_{2})^{n}(x_i, x_j) = \vee_{x_i x_j \epsilon V}\{(\mu_2{}^{e})^{n-1}(x_i, x_j) \times \mu_2{}^{e}(x_i, x_j)\}$

3.11. Definition. (Complement of product IFSG). The complement of PIFSG is $PIG(A, V^{C}, (V \times V)^{C})$where $V^{C} = ((\rho_1{}^{e})^{C}, (\rho_2{}^{e})^{C})and\ (V \times V)^{C} = (\mu_1{}^{e})^{C}, (\mu_2{}^{e})^{C}$. Here $(\rho_1{}^{e})^{C} = (\rho_2{}^{e})^{C}$ and $(\mu_1{}^{e})^{C} = (\mu_2{}^{e})^{C}$and $(\mu^{e}{}_{1})^{C}(x_i, x_j) = \rho_1{}^{e}(x_i) \times \rho_1{}^{e}(x_j) - \mu^{e}{}_{1}(x_i, x_j)$. Also, $(\mu^{e}{}_{2})^{C}(x_i, x_j) = \rho_2{}^{e}(x_i) \times \rho_2{}^{e}(x_j) - \mu^{e}{}_{2}(x_i, x_j)$.

In the next section, we introduce a new structure name as Multi intutionistic soft fuzzy graph structure.

4. Coclusion

In this paper , an attempt to generalize Fuzzy graph to Intutionistic soft fuzzy graph is made. Graph theory and Fuzzy Sets are related by using various structures.

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