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A SPARRE ANDERSEN DEPENDENT RISK MODEL WITH CONSTANT INTEREST FORCE

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Abstract: In this paper we consider a Sparre Andersen dependent risk model affected by a constant interest force. In this dependent model the distribution of inter-arrival time of claims depends on previous claim size. Here we derived integro-differential equations for the survival probabilities. As a special case differential equations, where the claim size follows Erlang distribution, were also derived.

Key Words: Survival probability; Sparre Andersen model; Constant interest force; integro-differential equation; Erlang distribution.

1. Introduction

The classical risk models relies on three major assumptions

- Claim occur as a Poisson process
- Claim size and inter-arrival time are independent
- Surplus receives no interest

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By violating the first assumption Andersen (1957) modified the claim occurrence process to a more general renewal process. Since then Sparre Andersen models are often candidates for much of the insurance studies. Dickson (1998) studied a renewal process in which claims occur as an Erlang process. Various aspects of Erlang process were showed in Dickson and Hipp (1998), Dickson and Hipp (2001), Cheng and Tang (2003), Sun and Yang (2004), Li and Garrido (2004), Tsai and Sun (2004), Gerber and Shiu (2005), Albrecher, Claramunt, and Marmol (2005), Marmol, Bielsa, and Lacayo (2005), Li and Dickson (2006) and Li (2008).

The assumption of independence is too restrictive in practice. To avoid this restriction, the effect of dependence was studied by Nyrhinen (1998), Nyrhinen (1999), Mu¨ller and Pflug (2001), Yuen and Guo (2001), Yuen, Guo, and Wu (2002), Albrecher and Kantor (2002), Asmussen and Højgaard (1999), Albrecher and Boxma (2004), Albrecher and Teugels (2006), and Albrecher, Constantinescu, and Loisel (2011). Kwan and Yang (2007) reversed the dependence structure of the model developed by Albrecher and Boxma (2004) and it's another aspect was considered by Meng, Zhang, and Guo (2008). Most of the risk models were developed under the assumption of absence of investment income. But in practice major part of the surplus is dedicated by investment income. In recent years the impact of interest income was carefully examined by Sundt and Teugels (1995), Sundt and Teugels (1997), Paulsen and Gjessing (1997), Kalashnikov and Norberg (2002), Yang and Zhang (2003), Cai (2002), Cai and Dickson (2003), Kalashnikov and Konstantinides (2000), Thampi and Jacob (2013), and Cardoso and Waters (2003). In this study we combine all three aspects to a single model.

2. The Model

Let us consider the following surplus process

$$U_\delta(t) = ue^{\delta t} + c\bar{s}_t^{(\delta)} - \int_0^t e^{\delta(t-v)} dS(v), t \geq 0$$

where $U_\delta(t)$ is the insurer's surplus at time t

$u = U_\delta(0)$ is the initial surplus

δ is the constant interest force

c is the constant premium rate

S(v) is the accumulated claim upto time v

and

$$\bar{s}_t^{(\delta)} = \int_0^t e^{\delta v} d_v \begin{cases} = t & if \quad \delta = 0 \\ = \frac{e^{\delta t}-1}{\delta} & if \ \delta > 0 \end{cases} \tag{1}$$

Let $\{X_i\}_{i=1}^{\infty}$ be a sequence of independent and identically distributed random variables with mean μ where X_i denote the size of the i^{th} claim. Also let N(t) be the number of claims upto time t. Then $S(v) = \sum_{i=1}^{N(v)} X_i$

Let us assume the following Markovian dependence structure of claim occurrence process: if claim size X_i is larger than a threshold A_i then T_{i+1} follows Erlang(2) distribution with parameter β_1 otherwise it follows Erlang(2) distribution with parameter β_2.

The net profit condition of this dependent model is

$$\mu < 2c\left(\frac{p(A \leq X)}{\beta_1} + \frac{p(A > X)}{\beta_2}\right) \tag{2}$$

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Let $T = inf_{t\geq 0}\{t: U_\delta(t) < 0\}$ be the time of ruin then

$$\phi_{\delta i}(u) = p[T = \infty / U(0) = u, T_i \sim Erlang(2, \beta_i)] \quad (3)$$

be the ultimate survival probability under constant interest force δ given the first claim occurs according to the Erlang distribution with rate $\beta_i, (i = 1,2)$.

2.1 Theorem. The ultimate survival probability $\phi_{\delta i}(u), i = 1,2,$ satisfies the following integro-differential equation

$$\frac{d^2}{d_u^2}\phi_{\delta i}(u) + \left(\frac{\delta - 2\beta_i}{c + \delta u}\right)\frac{d}{d_u}\phi_{\delta i}(u) + \left(\frac{\beta_i}{c + \delta u}\right)^2\phi_{\delta i}(u) =$$

$$\left(\frac{\beta_i}{c+\delta u}\right)^2 \int_0^u [p(A \leq X)\phi_{\delta 1}(u - x)f(x) + p(A > X)\phi_{\delta 2}(u - x)f(x)]d_x \quad (4)$$

Proof:- By conditioning on the time and the amount of the first claim and for i = 1 we have

$$\phi_{\delta 1}(u) = E\{\phi_{\delta 1}U(T_1)\} = E\phi_{\delta 1}(ue^{\delta T_1} + c\bar{s}_t(\delta) - X_1)$$

$$\phi_{\delta 1}(u) = \int_0^\infty \beta_1^2 te^{-\beta_1 t} \int_0^{ue^{\delta t}+c\bar{s}_t(\delta)} [p(A \leq X)\phi_{\delta 1}(ue^{\delta t} + c\bar{s}_t(\delta) - x)f(x) +$$
$$P(A > x)\phi_{\delta 2}(ue^{\delta t} + c\bar{s}_t(\delta) - x)f(x)]d_x d_t$$

By changing $s = ue^{\delta t} + c\bar{s}_t(\delta)$

$$\phi_{\delta 1}(u) = \int_u^\infty \frac{\beta_1^2}{\delta} log\left(\frac{c + \delta s}{c + \delta u}\right)\left(\frac{c + \delta u}{c + \delta s}\right)^{\frac{\beta_1}{\delta}} \frac{d_s}{c + \delta s}\int_0^s [p(A \leq X)\phi_{\delta 1}(s - x)f(x)$$
$$+ p(A > X)\phi_{\delta 2}(s - x)f(x)]d_x$$

$$= \int_u^\infty \frac{\beta_1^2}{\delta}(c + \delta u)^{\frac{\beta_1}{\delta}}(c + \delta s)^{-\frac{\beta_1}{\delta}-1} log\left(\frac{c+\delta s}{c+\delta u}\right)\int_0^\infty p(A \leq X)\phi_{\delta 1}(s - x)f(x) + p(A > X)\phi_{\delta 2}(s - x)f(x)d_x d_s \quad (4)$$

Differentiating (5) with respect to u we get

$$\frac{d}{d_u}\phi_{\delta 1}(u) = \int_u^{\infty} \frac{\beta_1^3}{\delta}(c+\delta s)^{-\frac{\beta_1}{\delta}-1}(c+\delta u)^{\frac{\beta_1}{\delta}-1} log(\frac{c+\delta s}{c+\delta u})$$

$$\int_0^s [p(A \leq X)\phi_{\delta 1}(s-x)f(x) + p(A > X)\phi_{\delta 2}(s-x)f(x)]d_x d_s$$

$$-\int_u^{\infty} \beta_1^2(c+\delta s)^{-\frac{\beta_1}{\delta}-1}(c+\delta u)^{\frac{\beta_1}{\delta}-1}\int_0^s [p(A \leq X)\phi_{\delta 1}(s-x)f(x)$$

$$+ p(A > X)\phi_{\delta 2}(s-x)f(x)]d_x d_s$$

$$= \beta_1(c+\delta u)^{-1}\phi_{\delta 1}(u) - \int_u^{\infty} \beta_1^2(c+\delta s)^{-\frac{\beta_1}{\delta}-1}(c+\delta u)^{\frac{\beta_1}{\delta}-1}[p(A \leq X)\phi_{\delta 1}(s-x)f(x) + p(A > X)\phi_{\delta 2}(s-x)f(x)]d_x d_s \quad (6)$$

Differentiating again with respect to u we get

$$\frac{d^2}{d_u^2}\phi_{\delta 1}(u) + \beta_1\delta(c+\delta u)^{-2}\phi_{\delta 1}(u) - \beta_1(c+\delta u)^{-1}\frac{d}{d_u}\phi_{\delta 1}(u)$$

$$= \beta_1^2(c+\delta u)^{-2}\int_0^u [p(A \leq X)\phi_{\delta 1}(s-x)f(x) + p(A > X)\phi_{\delta 2}(s-x)f(x)]d_x +$$

$$\delta(\frac{\beta_1}{\delta} - 1)(c+\delta u)^{-1}\frac{d}{d_u}\phi_{\delta 1}(u) - \beta_1(c+\delta u)^{-1}\phi_{\delta 1}(u) \quad (7)$$

$$\frac{d^2}{d_u^2}\phi_{\delta 1}(u) - \frac{d}{d_u}\phi_{\delta 1}(u)\delta(\frac{\beta_1-\delta}{\delta})(c+\delta u)^{-1} + \beta_1(c+\delta u)^{-1}$$

$$+\phi_{\delta 1}(u)\beta_1\delta(c+\delta u)^{-2} + \delta(\frac{\beta_1-\delta}{\delta})(c+\delta u)^{-1}\beta_1(c+\delta u)^{-1}$$

$$= \beta_1^2(c+\delta u)^{-2}\int_0^u [p(A \leq X)\phi_{\delta 1}(s-x)f(x) + p(A > X)\phi_{\delta 2}(s-x)f(x)]d_x$$

$$\frac{d^2}{d_u^2}\phi_{\delta 1}(u) + (\frac{\delta - 2\beta_1}{c+\delta u})\frac{d}{d_u}\phi_{\delta 1}(u) + (\frac{\beta_1}{c+\delta u})^2\phi_{\delta 1}(u)$$

$$= (\frac{\beta_1}{c+\delta u})^2 \int_0^u p(A \leq X)\phi_{\delta 1}(u-x)f(x) + p(A > X)\phi_{\delta 2}(u-x)f(x)d_x \quad (8)$$

Similarly for i = 2

$$\frac{d^2}{d_u^2}\phi_{\delta 2}(u) + (\frac{\delta - 2\beta_2}{c+\delta u})\frac{d}{d_u}\phi_{\delta 2}(u) + (\frac{\beta_2}{c+\delta u})^2\phi_{\delta 2}(u)$$

$$= (\frac{\beta_2}{c+\delta u})^2 \int_0^u p(A \leq X)\phi_{\delta 1}(u-x)f(x) + p(A > X)\phi_{\delta 2}(u-x)f(x)d_x \quad (9)$$

For the special case if we take $\delta = 0$ we obtain from **Error! Reference source not found.**

$$c^2 \frac{d^2}{d_u^2}\phi_i(u) - 2\beta_i c \frac{d}{d_u}\phi_i(u) + \beta_i^2 \phi_i(u)$$

$$= \beta_i^2 \int_0^u P(A \le x) f(x)\phi_1(u-x) + P(A > x) f(x)\phi_2(u-x) d_x \qquad (10)$$

which is the integro-differential equation given in Sajithamony and Thampi (2015) for the Markov Dependent Sparre Andersen Risk model without investment income.

3. Erlang(2) Claim Size Distribution

Suppose the distribution of claim size follows Erlang distribution, $f(x) = \lambda^2 x e^{-\lambda x}$ and the random threshold variable A follows Erlang distribution, $f(A) = \theta^2 A e^{-\theta A}$

In this section we insert for f(x) and f(A) to find the differential equation.

3.1 Theorem. $\phi_{\delta i}(u), i = 1,2$ satisfies the following systems of differential equations

$$(c+\delta u)^2 \frac{d^4}{d_u^4}\phi_{\delta 1}(u) + (c+\delta u)(5\delta - 2\beta_1) + 2\lambda(c+\delta u)^2 \frac{d^3}{d_u^3}\phi_{\delta 1}(u) + (\beta_1^2 - 4\delta(\delta - \beta_1)) + 2\lambda(c+\delta u)(3\delta - 2\beta_1) + \lambda^2 (c+\delta u)^2 \frac{d^2}{d_u^2}\phi_{\delta 1}(u) + 2\lambda(\beta_1^2 + \delta(\delta - 2\beta_1)) + \lambda^2(c+\delta u)(\delta - 2\beta_1)\frac{d}{d_u}\phi_1(u) = \lambda^2 \beta_1^2 \frac{\lambda^2(3\theta+\lambda)}{(\theta+\lambda)^3}\phi_{\delta 2}(u) - \phi_{\delta 1}(u) \qquad (11)$$

$$(c+\delta u)^2 \frac{d^4}{d_u^4}\phi_{\delta 2}(u) + (c+\delta u)(5\delta - 2\beta_2) + 2\lambda(c+\delta u)^2 \frac{d^3}{d_u^3}\phi_{\delta 2}(u) + (\beta_2^2 - 4\delta(\delta - \beta_2)) + 2\lambda(c+\delta u)(3\delta - 2\beta_2) + \lambda^2 (c+\delta u)^2 \frac{d^2}{d_u^2}\phi_{\delta 2}(u) + 2\lambda(\beta_2^2 + \delta(\delta - 2\beta_2)) + \lambda^2(c+\delta u)(\delta - 2\beta_2)\frac{d}{d_u}\phi_2(u) = \lambda^2 \beta_2^2 \frac{\theta^2(\theta+3\lambda)}{(\theta+\lambda)^3}\phi_{\delta 1}(u) - \phi_{\delta 2}(u) \qquad (12)$$

Proof:- For i = 1 we have

$$(c+\delta u)^2 \frac{d^2}{d_u^2}\phi_{\delta 1}(u) + (\delta - 2\beta_1)(c+\delta u)\frac{d}{d_u}\phi_{\delta 1}(u) + \beta_1^2 \phi_{\delta 1}(u)$$

$$= \beta_1^2 \int_0^u [P(A \le X)\phi_{\delta 1}(u-x)f(x) + P(A > X)\phi_{\delta 2}(u-x)f(x)]d_x \qquad (13)$$

Now define $h_1(u) = \beta_1^2 \int_0^u [P(A \le u-x)f(u-x)\phi_{\delta 1}(x) + P(A > u-x)f(u-x)\phi_{\delta 2}(x)]d_x$

Let $A \sim Erlang(2,\theta)$ and $X \sim Erlang(2,\lambda)$ then

$$h_1(u) = \beta_1^2 \int_0^u \{\frac{(\theta+3\lambda)^2}{(\theta+\lambda)^3}\lambda^2(u-x)e^{-\lambda(u-x)}\phi_{\delta 1}(x) + \frac{\lambda^2(3\theta+\lambda)}{(\theta+\lambda)^3}\lambda^2(u-x)e^{-\lambda(u-x)}\phi_{\delta 2}(x)\}d_x$$

Differentiating with respect to u we get

$$\frac{d}{d_u}h_1(u) = \beta_1^2 \int_0^u \lambda^2\theta^2\frac{(\theta+3\lambda)}{(\theta+\lambda)^3}\phi_{\delta 1}(x) + \frac{\lambda^2(3\theta+\lambda)}{(\theta+\lambda)^3}\phi_{\delta 2}(x)(u-x)e^{-\lambda(u-x)}(-\lambda) + e^{-\lambda(u-x)}d_x$$

$$= -\lambda h_1(u) + \beta_1^2\lambda^2 e^{-\lambda u}\int_0^u e^{\lambda x}\frac{\theta^2(\theta+3\lambda)}{(\theta+\lambda)^3}\phi_{\delta 1}(x) + \frac{\lambda^2(3\theta+\lambda)}{(\theta+\lambda)^3}\phi_{\delta 2}(x)d_x$$

Differentiating again with respect to u we get

$$\frac{d^2}{d_u^2}h_1(u) = -\lambda\frac{d}{d_u}h_1(u) - \beta_1^2\lambda^3 e^{-\lambda u}\int_0^u e^{\lambda x}\frac{\theta^2(\theta+3\lambda)}{(\theta+\lambda)^3}\phi_{\delta 1}(x) + \frac{\lambda^2(3\theta+\lambda)}{(\theta+\lambda)^3}\phi_{\delta 2}(x)d_x + \beta_1^2\lambda^2\frac{\theta^2(\theta+3\lambda)}{(\theta+\lambda)^3}\phi_{\delta 1}(u) + \frac{\lambda^2(3\theta+\lambda)}{(\theta+\lambda)^3}\phi_{\delta 2}(u)$$

$$= -\lambda\frac{d}{d_u}h_1(u) - \lambda\frac{d}{d_u}h_1(u) + \lambda h_1(u) + \beta_1^2\lambda^2\frac{\theta^2(\theta+3\lambda)}{(\theta+\lambda)^3}\phi_{\delta 1}(u) + \frac{\lambda^2(3\theta+\lambda)}{(\theta+\lambda)^3}\phi_{\delta 2}(u)$$

$$\frac{d^2}{d_u^2}h_1(u) + 2\lambda\frac{d}{d_u}h_1(u) + \lambda^2 h_1(u) = \lambda^2\beta_1^2\frac{\theta^2(\theta+3\lambda)}{(\theta+\lambda)^3}\phi_{\delta 1}(u) + \frac{\lambda^2(3\theta+\lambda)}{(\theta+\lambda)^3}\phi_{\delta 2}(u) \qquad (6)$$

Thus we can eliminate the integral term from (3) using (6)

$$(c+\delta u)^2 \frac{d^4}{d_u^4}\phi_{\delta 1}(u) + (c+\delta u)(5\delta - 2\beta_1)\frac{d^3}{d_u^3}\phi_{\delta 1}(u) + (\beta_1^2 + 4\delta(\delta - \beta_1))\frac{d^2}{d_u^2}\phi_1(u)$$

$$+2\lambda(c+\delta u)^2 \frac{d^3}{d_u^3}\phi_{\delta 1}(u) + (c+\delta u)(3\delta - 2\beta_1)\frac{d^2}{d_u^2}\phi_{\delta 1}(u) + (\beta_1^2 + \delta(\delta - 2\beta_1))\frac{d}{d_u}\phi_{\delta 1}(u)$$

$$+\lambda^2(c+\delta u)^2 \frac{d^2}{d_u^2}\phi_{\delta 1}(u) + (c+\delta u)(\delta - 2\beta_1)\frac{d}{d_u}\phi_{\delta 1}(u) + \beta_1^2\phi_{\delta 1}(u)$$

$$= \lambda^2\beta_1^2 \frac{\theta^2(\theta+3\lambda)}{(\theta+\lambda)^3}\phi_{\delta 1}(u) + \frac{\lambda^2(3\theta+\lambda)}{(\theta+\lambda)^3}\phi_{\delta 2}(u)$$

$$(c+\delta u)^2 \frac{d^4}{d_u^4}\phi_{\delta 1}(u) + (c+\delta u)(5\delta - 2\beta_1) + 2\lambda(c+\delta u)^2\frac{d^3}{d_u^3}\phi_{\delta 1}(u) + (\beta_1^2 + 4\delta(\delta - \beta_1)) + 2\lambda(c+\delta u)(3\delta - 2\beta_1) + \lambda^2(c+\delta u)^2\frac{d^2}{d_u^2}\phi_{\delta 1}(u) + 2\lambda(\beta_1^2 + \delta(\delta - 2\beta_1)) + \lambda^2(c+\delta u)(\delta - 2\beta_1)\frac{d}{d_u}\phi_{\delta 1}(u) + \lambda^2\beta_1^2 1 - \frac{\theta^2(\theta+3\lambda)}{(\theta+\lambda)^3}\phi_{\delta 1}(u)$$

$$= \lambda^2\beta_1^2 \frac{\lambda^2(3\theta+\lambda)}{(\theta+\lambda)^3}\phi_{\delta 2}(u)$$

$$(c+\delta u)^2 \frac{d^4}{d_u^4}\phi_{\delta 1}(u) + (c+\delta u)(5\delta - 2\beta_1) + 2\lambda(c+\delta u)^2\frac{d^3}{d_u^3}\phi_{\delta 1}(u)$$

$$+(\beta_1^2 + 4\delta(\delta - \beta_1)) + 2\lambda(c+\delta u)(3\delta - 2\beta_1) + \lambda^2(c+\delta u)^2\frac{d^2}{d_u^2}\phi_{\delta 1}(u)$$

$$+2\lambda(\beta_1^2 + \delta(\delta - 2\beta_1)) + \lambda^2(c + \delta u)(\delta - 2\beta_1)\frac{d}{d_u}\phi_{\delta 1}(u) =$$

$$\lambda^2\beta_1^2\frac{\lambda^2(3\theta+\lambda)}{(\theta+\lambda)^3}\phi_{\delta 2}(u) - \phi_{\delta 1}(u) \tag{7}$$

Similarly for i = 2 we have

$$(c + \delta u)^2\frac{d^4}{d_u^4}\phi_{\delta 2}(u) + (c + \delta u)(5\delta - 2\beta_2) + 2\lambda(c + \delta u)^2\frac{d^3}{d_u^3}\phi_{\delta 2}(u)$$

$$+(\beta_2^2 + 4\delta(\delta - \beta_2)) + 2\lambda(c + \delta u)(3\delta - 2\beta_2) + \lambda^2(c + \delta u)^2\frac{d^2}{d_u^2}\phi_{\delta 2}(u)$$

$$+2\lambda\left(\beta_2^2 + \delta(\delta - 2\beta_2)\right) + \lambda^2(c + \delta u)(\delta - 2\beta_2)\frac{d}{d_u}\phi_{\delta 2}(u) =$$

$$\lambda^2\beta_2^2\frac{\theta^2(\theta+3\lambda)}{(\theta+\lambda)^3}\phi_{\delta 1}(u) - \phi_{\delta 2}(u) \tag{16}$$

Using suitable methods of solving system of higher order differential equations we can find the survival probabilities $\phi_{\delta 1}(u)$ and $\phi_{\delta 2}(u)$.

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